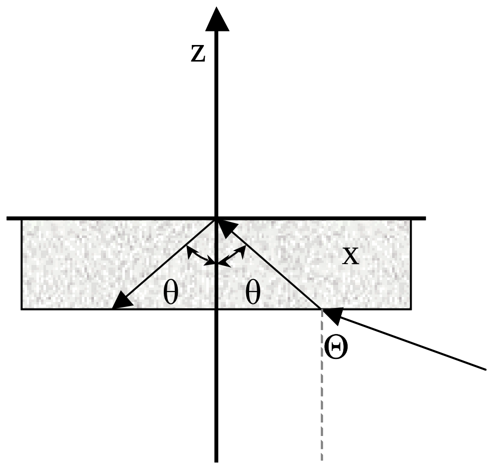
Holographic optical elements: The notch filter

"A notch rejection filter (i.e. that blocks light in a narrow band of wavelengths) can be made as a holographic optical element (HOE). To make such a filter, a laser beam refracted at an angle θ within the holographic emulsion is reflected off a mirror contacted to its back surface to produce interference fringes parallel to the surface. Using a laser at 578 nm to make the hologram, what angle of incidence is necessary to produce a filter that will reject the 694 nm light of a HeNe laser at normal incidence? Take the refractive index of the emulsion as being n = 1.45. Explain any steps in the argument and/or calculations."



Beam is incident on the emulsion at Θ to the normal (z-axis), and propagates at θ to the normal through the emulsion

Reflection off the mirror may be considered to be another beam, also propagating at θ to the normal while in the emulsion.

The interference pattern created by these waves will contain intensity maxima that propagate along the x-axis, in the average of the two beam directions.

When the intensity of the superposition of these waves is time-averaged, a standing wave will result, parallel to the x-axis (emulsion surface).

Taking the beam and its reflection (both within the emulsion) to be plane waves:

$$E_{incoming}(\vec{r},t) = E_1 = \cos\left(\vec{k}\cdot\vec{r} - \omega t\right)$$
$$E_{reflect}(\vec{r},t) = E_2 = \cos\left(\vec{k}^*\cdot\vec{r} - \omega t\right)$$

Using $k = \left|\vec{k}\right|$:

$$\vec{k} = k\hat{x}\sin(\theta) + k\hat{z}\cos(\theta)$$
$$\vec{k}^* = k\hat{x}\sin(\theta) - k\hat{z}\cos(\theta)$$

Linear superposition of the two waves, with z component separated:

$$\begin{aligned} E(x,z,t) &= E(\vec{r},t) = E_1 + E_2 \\ &= \cos(kx\sin\theta - \omega t + kz\cos\theta) + \cos(kx\sin\theta - \omega t - kz\cos\theta) \\ &= 2\cos(kx\sin\theta - \omega t)\cos(kz\cos\theta) \end{aligned}$$

Time-average to obtain an intensity, which doesn't depend on x (due to x and *t* appearing as a sum in the parameter to a periodic function)

$$\begin{aligned} I(z) &\propto \left\langle E^2(x,z,t)\right\rangle = \left\langle \cos^2(kx\sin\theta - \omega t)\cos^2(kz\cos\theta)\right\rangle \\ &\propto \cos^2(kz\cos\theta) \propto 1 + \cos(2kz\cos\theta) \\ &= 1 + \cos\left(\frac{4\pi\cos\theta}{\lambda}z\right) \\ &= 1 + \cos\left(\frac{2\pi}{\Lambda}z\right) \end{aligned}$$

As we need the grating to have the same period as the filtered light's E-field (not intensity field), a factor of a half arises in the left-hand-side of the grating spacing calculation:

$$\therefore \frac{\Lambda}{2} = \frac{\lambda}{2\cos\theta} \quad \rightarrow \quad \Lambda = \lambda / \cos\theta$$

$$\therefore \theta = \cos^{-1}\left(\lambda / \Lambda\right)$$

The wavelengths of the beams will be different in the emulsion, due to its refractive index being different to that of the surrounding medium (assumed to be air):

Using:

$\lambda = n\lambda' = n \times 578$ nm — λ' is the wavelength (in air) of light used to make the grating

$\Lambda = n\Lambda' = n \times 694$ nm — Λ' is the centre wavelength (in air) of the notch filter

Where:

$n = n_{air} / n_{emulsion}$ — The ratio of refractive indices at the air-emulsion interface

$\theta = \cos^{-1}\left(578n / 694n\right) = 33.6°$ — The angle (in the emulsion) of the recording beam

The beam must travel through the emulsion at 33.6° to the surface normal.

Applying Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\Theta = \frac{n_{emulsion} \sin\theta}{n_{air}}$$

$$\Theta = \sin^{-1}\left(\frac{n_{emulsion} \sin\theta}{n_{air}}\right) = \sin^{-1}\left(1.45\sin(33.6°)\right)$$

$$\Theta = 53.4°$$

The beam must be angled at 53.4° to the normal of the plate.