Underwater vision and f-numbers of the eye

Underwater Vision

"Show by calculation that when you swim underwater (without goggles) your vision will become blurry. Explain the steps in your argument. Write a sentence or two that explains the (optical physics) reason for blurry vision. Marks will be allocated for a fully explained answer with appropriate diagram(s)."

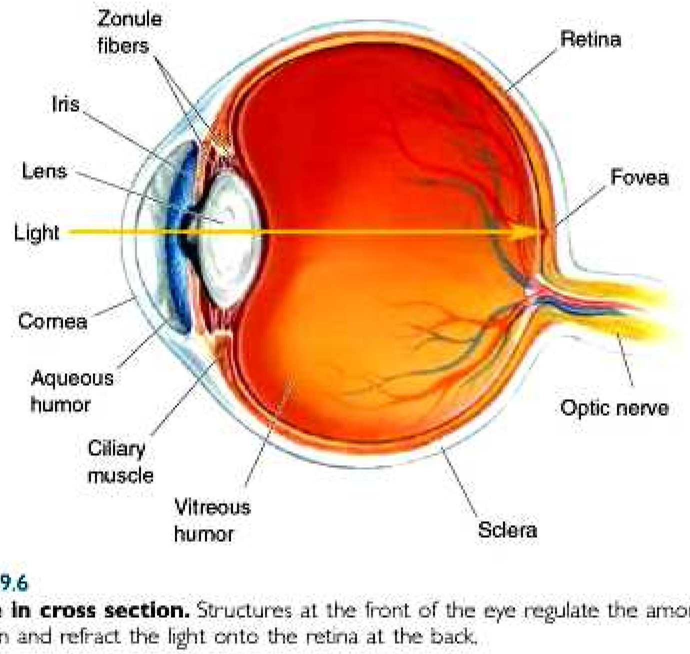

Figure 1: Anatomy of the eye (copied from source A)
Anterior surfaces face leftwards, posterior face rightwards.

Contrary to expectations, most of the focusing power in the eye is with the cornea, not the lens. Typically, the focusing power of the cornea is around 42 dioptres[A,B]. The lens merely provides corrective "fine-tuning" and accommodation for non-paraxial rays from sources close to the eye. The cause of this behaviour may be explored by considering refractive indices[C] at different sections of the optical path from image to retina (all for ~589 nm light):

- Air (outside the eye) ___ 1.000
- Cornea _______________ 1.34 - 1.38 (depending on model used)
- Aqueous humour______ 1.336 – 1.337
- Lens __________________ 1.42 - 1.43 (its shape changes, allowing variable focal power)
- Vitreous humour_______ 1.336
- Water ________________ 1.333

For an asymmetric spherical lens of refractive index n_2 preceded by medium of refractive index n_1 and succeeded by a medium of refractive index n_3 the focusing power is given by the Lens Maker's equation[D]:

$$P = \frac{1}{n_3}\left(\frac{n_2 - n_1}{R_1} - \frac{n_3 - n_2}{R_2}\right)$$

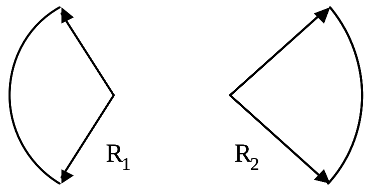

Figure 2: Lens geometry for diagram on right

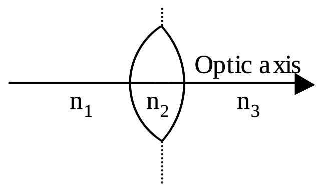

Figure 3: Refractive indices for above equation

Assuming cornea radii[C] of $R_1 = 7.8\ \text{mm}$ and $R_2 = 6.5\ \text{mm}$ (Figure 2) and maximal values (from above table) for refractive indices, the focusing power of an air–cornea–aqueous humour section (Figure 3) is approximately $41\ \text{dioptres}$. Repeating this calculation, with the air replaced by water, gives a focusing power of $9.5\ \text{dioptres}$, considerably lower than the power calculated for air. Remembering that the focal power is the reciprocal of the focal length – this change results in an underwater focal length of $11\ \text{cm}$, considerably larger than a human eyeball.

Can the lens correct this? According to source C, the lens can provide additional focusing power of up to ten dioptres. This is not sufficient to correct the loss of thirty dioptres, caused by the higher refractive index of water (compared to air) at the anterior surface of the cornea.

[A] NEUROSCIENCE: Exploring the Brain (Third Edition), MF Bear, BW Connors, MA Paradiso, *ISBN: 978-0-7817-6003-4*

[B] Dioptre (reciprocal metre) = $1/\text{focal length}$, giving the expected focal length for a typical cornea of approximately 24 mm.

[C] "The Eye", Lance Hahn, http://retina.anatomy.upenn.edu/~lance/eye/eye.html acquired 08-03-2011
Author credentials may be found at http://retina.anatomy.upenn.edu/~lance/l_w_h.html

[D] "Lens Maker's Formula", http://www.tutorvista.com/physics/lens-makers-formula acquired 08-03-2011

The human eye's range of f-numbers

"Calculation and 2 – 3 sentences. Make an estimate of the range of f# numbers possible in the human eye. Explain how you arrived at your answer."

Using the information and sources from the previous question, the focal power of a typical human eye is composed of the cornea focal power and the additional variable focusing from the lens. The focal power of the cornea was calculated to be approximately $41\ \text{dioptres}$, and the power of the lens is given as a range from zero to ten dioptres.

The f-number is calculated by $F = f/D = 1/PD$, where:

F is the f-number

f is the focal length

D is the diameter of the pupil aperture

P is the focal power ($f = 1/P$)

The maximal f-number therefore occurs at minimal values of *P* and *D* (vice versa). Assuming a typical pupil diameter ranges from $2\ \text{mm to } 8\ \text{mm}$[A], and maintaining the assumption that the lens provides up to ten dioptres or focal power, the following f-numbers are obtained:

- $P = 41$ dioptres; $D = 2$ mm; $F_{max} = 12$
- $P = 51$ dioptres; $D = 8$ mm; $F_{max} = 2.5$

That is to say, the f-numbers for a typical human eye range from approximately f/2.5 at the "fast" end of the scale to f/12 at the "slow" end. The average cheap digital camera will support f-numbers ranging from f/2.8 to around f/16 (at full zoom).

[A] "The Reaction of the Eye to Light", P Reeves, *Transactions of the Optical Society, Vol. 22, No. 1*, pp. 1-14 (1920)